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DETERMINATION OF GAS PARAMETERS IN RESONANT PIPES AND CHANNELS OF ENGINES WITH A PERIODIC WORKFLOW USING THE PISTON ANALOGY METHOD

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This paper investigates a process of gas flow in the resonant tube of an engine with a periodic workflow. Analysis of various flow models and comparison of known data have shown that the problems of constructing closed 0-dimensional models of the operating cycle for some types of engines remain unresolved. Given this, the question arises about the dimensionality of models of individual engine elements, including the resonant pipe model, which must be included in the general model of the cycle, especially at the initial stage of its development.

To solve the identified problems, a mathematical model of air flow has been improved, built on the basis of an analogy with a ‘”liquid” piston. Unlike existing ones, the piston analogy model allows one to calculate the instantaneous velocity averaged over the length of the pipe using a numerical solution of the differential equation for velocity.

To test the model built, an alternative finite-difference 1-dimensional gas-dynamic model was selected, with the help of which a test simulation of air flow in a pipe was performed. It has been established that the piston model allows one to find the flow velocity with an accuracy of 5 % for a pressure drop varying according to a sinusoidal law. The permissible limits for changes in the oscillation frequency and pipe length were found, at which the piston model has a minimum error.

Based on the results of the study, it was concluded that with a small mass and inertia of the liquid piston, the proposed model gives results close to those provided by more complex models with higher dimensionality. This indicates the possibility of using a piston model for elements such as pipes as part of a 0-dimensional thermodynamic model of engines with a periodic operating process as an approximate alternative to traditional 1-dimensional flow models

Keywords: pulse jet engine, resonant tube, flow modeling, “liquid” piston method, piston analogy

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1. Introduction

It is known that in engines with a periodic workflow, oscillatory phenomena occur in internal channels and pipes, which have a significant impact on the operating process and output parameters of the engines. Such engines include all types of internal and external combustion engines (Stirling engines), as well as pulse jet engines.

To correctly “tune” the geometric characteristics of channels and pipes to certain operating modes, the so-called Cadenasi effect [1] is used, when the cylinder (combustion chamber) is dynamically recharged with air due to resonant pressurization. It is typical that this effect is used in both intake and exhaust systems [2]. Thanks to this, air flow through the engine increases, pressure in the cylinder (combustion chamber) increases while losses of unburned fuel into the exhaust system are reduced. This significantly improves all the main parameters of engines. Moreover, some engines (for example, a pulse jet) may not work at all if the dimensions are incorrectly selected and the resonance tubes are incorrectly configured [3].

In accordance with this, in practice the problem of modeling flow in resonant pipes often arises in order to select their optimal geometry. For this purpose, a large number of models and programs have been developed, including both one-dimensional (for example, the well-known method of characteristics [4]) and 2- and 3-dimensional [5] ones. Among them, one can note universal models that adapt to the type of engines in question and their elements. These can also be purely special models that are not only strictly tied to specific types of structures [6] but are also applicable mainly to existing prototypes of a given geometry.

At the same time, in existing flow models there is a clear tendency to gradually increase their complexity by increasing the dimensionality of the model, more accurately taking into account heat losses, flow regimes, and vortex formation. This is generally understandable since more complex models give more accurate results. At the same time, this makes complex models less suitable for use when developing new models of the operating cycle of engines of the type in question. On the contrary, simple flow models are more convenient for debugging programs but their use requires justification of the reliability of results. In this regard, it seems necessary to build an appropriate model and evaluate it by comparing the flow simulation results with more accurate known models.

Thus, the relevance of the study is determined by the need for further development of mathematical modeling and the construction of new models of engines of the type under consideration, including investigating, for this purpose, flow processes in resonant tubes under unsteady mode.

2. Literature review and problem statement

The problem is that any engine with a periodic workflow requires that the modeling of any of its elements be closed to the operating cycle and output parameters. In this type of engine, it is useless to try to model any elements separately from the engine itself. This distinguishes them from engines with a stationary flow (the working process of a stationary type includes, for example, gas turbine, ramjet, and rocket engines, in which pulsations are possible but are an anomaly). Nevertheless, attempts to apply universal models do occur, including to describe the flow in the pipes of engines with a periodic workflow, separately from considering the engine cycle [7]. However, it is difficult to call them successful. The reason is the absence of the engine itself in them, its established (after several initial cycles so that the influence of initial conditions disappears) working cycle and the process of creating usable work (power or thrust).

In fact, a completely different task arises – creating, first of all, a closed, i.e., capable of calculating several complete cycles, complete model of the entire engine with this element. For some engine types (not all), such models were built and, depending on the desired accuracy and complexity, these models can even be divided into several different types.

Thus, the most common are 0-dimensional thermodynamic models based on parameter values averaged over the volume, 1D models and 3D CFD models [6].

0-dimensional or thermodynamic models are based on a general description of the processes of intake-exhaust, mixing and combustion in a cylinder [8, 9]. Due to the averaging of instantaneous parameters, such models provide little detail of the processes, and some parameters can only be obtained experimentally. However, the use of 0-dimensional models can be very effective not only for preliminary research and creating the appearance of a prototype; with their help, it is also possible to study various types of engines or devices that have a volume or cylinder with an unsteady working process [10, 11].

If one adds 1-dimensional models of flow in channels adjacent to the volume to the thermodynamic model for volumes, then such a hybrid 0-1D model can become quite a serious tool for research. Such models form the basis of modeling, representing closed models of one type of engine with a periodic workflow. That is, these are full cycle models, where the final parameters of the previous cycle are the initial conditions for the subsequent cycle.

This is how the most famous models are built. However, some of them, for example, AVL-Boost (Austria) [12], which has a similar thermodynamic basis, provide for the possibility of improving individual elements (models), including in the direction of increasing their dimensionality. In the Ricardo-WAVE model (England) [13, 14], a one-dimensional representation of the flow in the inlet and outlet channels allows a 3-dimensional user model only for the mixing and combustion processes in the cylinder. The GT-Power program (USA) in some sources [15] is considered one of the most advanced, however, even in it, increasing the dimensionality is possible only for some models of in-cylinder processes. And the well-known LES program (Lotus Engine Simulation, England [16]) generally has a hard-wired algorithm, including a 1-dimensional channel model, and does not allow making any changes to the element models. There remains another common problem with these programs, related to the fact that these programs are not suitable for other types of engines with periodic workflow.

On the opposite side are 3D models (CFD), which are used for multidimensional modeling of flow and combustion in engines [6, 17]. This approach is complex, requires significant computational resources and long simulation times. In some cases, it is even necessary to reduce the dimensionality of the problem from 3D to 1D in some elements in order to reduce labor intensity and obtain an acceptable overall calculation time [18]. As a result, multidimensional CFD models are good for the final stages of a project, especially for determining the stress-strain state of parts [19] but are poorly suited for modeling engine cycles with a periodic workflow [20]. Especially for the initial stages of projects to create new engines, when studying engines of new and/or not common design schemes, to assess their performance, select the main dimensions, and form the appearance of the prototype.

For some engine designs with a periodic workflow, the situation is complicated by the fact that there are no proven standard models for them (pulse jet, Stirling). In some cases, this leads to not the most successful attempts to apply the most developed and complex 3D models of elements [21, 22]. For example, when it is still necessary to develop a general thermodynamic 0-dimensional model, and only then, based on it, try to improve the models of individual elements. Moreover, development must begin with low-dimensional elements and models, and only after they have been worked out, move on to more complex, including spatial problems. There are examples of such solutions, but they have not yet been developed and are limited to the simplest models [23], and usually no estimates of the accuracy of such models and discrepancies with one-dimensional models are given.

In other words, there remain unresolved problems of the correct choice of principles, modeling structure, and construction of closed 0-dimensional models of the operating cycle for certain types of engines with a periodic workflow [24]. Given this, the question arises about the dimensionality of models of individual engine elements that need to be included in the general cycle model, especially at the initial stage of its development.

3. The aim and objectives of the study

The purpose of this study is to evaluate the effectiveness of using simple models of gas flow in pipes adjacent to the control volume (working cylinder or combustion chamber). This will make it possible to use these models as components of 0-dimensional closed thermodynamic models of the operating cycle of engines with a periodic workflow.

To achieve this goal, it is necessary to solve the following tasks:

– to develop a piston analogy model for air flow in a pipe;

– to build an alternative 1-dimensional gas-dynamic model;

– to perform mathematical modeling of air flow in a pipe using both methods;

– to conduct a comparative analysis of the accuracy and reliability of the piston analogy model in relation to the type of engine under consideration, as well as assess the limits of its applicability.

4. The study materials and methods

The object of the study is the resonant pipe of an engine with a periodic workflow, through which gas flows from the control volume (cylinder, combustion chamber) into the environment.

Determining the gas flow velocity in a pipe used in modeling engines with a periodic workflow is possible by analytical and/or numerical integration of differential equations that describe this process. In a specific problem, a numerical-analytical method was chosen, when, using the adopted simplifying assumptions, the analytical integration of the equations along the coordinate (pipe length) was carried out, then a numerical solution was found for the differential equation with respect to the process time. The numerical modeling method in a one-dimensional statement was used as a test method to verify the results.

The initial data adopted for the modeling were the approximate geometric dimensions, their ratios, and characteristics of the elements of engines with a periodic workflow such as resonators, which are a control volume with an attached resonant tube [25].

The main hypothesis was the assumption that under certain conditions determined by the frequency of the process, the length of the pipe, and inlet parameters, the flow of gas in the pipe can be considered as the movement of a certain "liquid" piston [26]. In this case, the gas in the pipe is represented in the form of a certain body with mass and, accordingly, inertia.

To implement the analogy of a "liquid" piston, a number of additional simplifying assumptions have been adopted. The derivation of the estimation equations was carried out under the condition of averaging over the coordinates of the instantaneous gas velocity as it changes over time. Within the framework of the piston analogy concept, it is accepted that there are no mixing processes in the pipe of dissimilar gases having different temperatures. At the same time, it is allowed to attach to the "liquid" piston the mass from flowing into the pipe during the reverse flow of air with increased density (reduced temperature), as well as its subsequent expulsion from the pipe. These assumptions were adopted based on the results of a study of resonance tubes of certain types of engines with a periodic workflow [15, 27].

To construct a test 1-dimensional flow model and boundary conditions, known data on the nature of gas flow at the inlet and outlet of the pipes were used, as well as the experience of numerical modeling using finite-difference methods [28].

All calculations and modeling were performed in Excel with the XLfit extension (USA) [29] for constructing curves and calculating integrals (including the areas under the curves). The choice of the program was due to the limited volume (no more than 2000 time points per cycle) and the test nature of the calculations, as well as the lack of ready-made programs for solving the problem under consideration.

5. Results of investigating the piston analog model for air flow in a pipe

5. 1. Construction of a piston analogy model for air flow in a pipe

When an engine operates with a periodic workflow, an oscillatory process occurs in the pipes adjacent to the combustion chamber (Fig. 1), during which forced oscillations of gas parameters (pressure, temperature, and speed) occur in the pipe.

Flow with subsonic speeds and pressure differences was considered, for which it was assumed that at all points of the pipe the instantaneous pressure, temperature, and gas velocity are the same. This assumption is applicable for relatively short pipes and reduces the model under consideration to the so-called piston analogy of gas flow, that is, to the model of a "liquid" or "gas" piston [26, 30]. Then the movement of gas through the pipe can be considered as the movement of a column of gas of some mass (Fig. 1), which has the properties of inertia [27]. This means that under the influence of a time-varying pressure difference, the speed velocity of gas movement lags behind the pressure – approximately the same as when waves move in a pipe.

Thus, the solution to the problem was sought in the form of an equation for the velocity of the gas column in the pipe. It is important that the desired speed velocity is associated with a thermodynamic model that describes the state of the gas in the control volume to which the pipe is connected. Indeed, this volume V within the framework of a 0-dimensional representation can always be described by a system of 2 differential equations of the 1st order. For example, for gas temperature $\bar{T}$ and pressure $\bar{p}$ [24], such equations can be obtained from the equation of the first law of thermodynamics (energy) and the equation of state of the gas in the form:

$$\begin{cases} \dfrac{d\bar{T}}{d\bar{t}} = f_1\left(\bar{T}, \bar{p}, \dfrac{d\bar{m}}{d\bar{t}}, \dfrac{d\bar{Q}}{d\bar{t}}, V \ldots\right), \\ \dfrac{d\bar{p}}{d\bar{t}} = f_2\left(\bar{T}, \bar{p}, \dfrac{d\bar{m}}{d\bar{t}}, \dfrac{d\bar{Q}}{d\bar{t}}, V \ldots\right), \end{cases} \quad (1)$$

where $d\bar{m}/d\bar{t}$ is the mass flow of gas from or into the control volume (through the pipe), $d\bar{Q}/d\bar{t}$ is the rate of heat release in the volume (in the 1st approximation, when deriving the model equations, thermal processes were not considered and not taken into account).

The system of equations (1) includes the flow rate of gas flowing out or flowing into the volume under consideration, and it is directly determined by the speed velocity. Consequently, the equation for the velocity in the pipe obtained as a result of solving the problem of a "liquid" piston closes the 0-dimensional thermodynamic model.

When solving the problem, the estimation equations were written in dimensionless form. For this purpose, dimensionless variables were used: pressure $p = \bar{p}/\bar{p}_0$, temperature $T = \bar{T}/\bar{T}_0$, speed velocity $v = \bar{v}/\bar{v}_0$, time $t = \bar{t}\bar{a}_0/L$, coordinate $x = \bar{x}/L$, where $\bar{p}_0$, $\bar{T}_0$ – pressure and ambient temperature; $\bar{a}_0 = \sqrt{k\bar{R}_0\bar{T}_0}$ – speed of sound velocity in the environment; L is the characteristic size of the engine. In addition, some other quantities were written down in dimensionless form and were used later: the heat capacity of a gas at constant pressure $C_p = \bar{C}_p/\bar{C}_{p0}$, ; the gas constant $R = \bar{R}/\bar{R}_0$, gas density $\rho = \bar{\rho}/\bar{\rho}_0$.

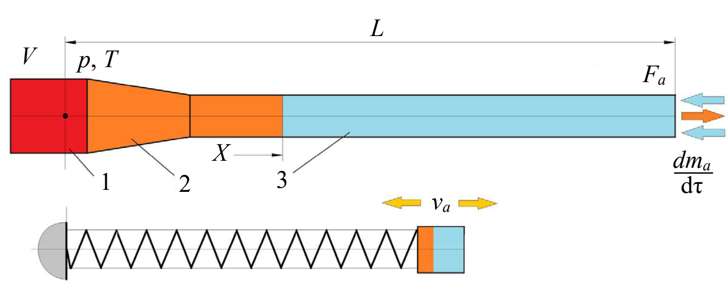

Fig. 1. Calculation diagram of the piston analogy method (liquid piston): 1 – control volume; 2 – adapter; 3 – resonant tube

Next, equations describing gas-dynamic processes in the pipe were considered [28]. This, among other things, is the continuity equation:

$$\frac{\partial\bar{\rho}}{\partial\bar{t}}+\frac{\partial}{\partial\bar{x}}\left(\overline{\rho v}\right)=0\ ,\ \frac{\partial\bar{\rho}}{\partial\bar{t}}+\frac{\partial}{\partial\bar{x}}\left(\overline{\rho v}\right)=0 \tag{2}$$

and the equation of motion (conservation of momentum):

$$\frac{\partial\bar{v}}{\partial\bar{t}}+\bar{v}\frac{\partial\bar{v}}{\partial\bar{x}}=-\frac{1-\delta}{\bar{\rho}}\frac{\partial\bar{p}}{\partial\bar{x}}\ , \tag{3}$$

where δ is the portion of the longitudinal pressure gradient spent on friction and local resistance.

Friction and hydraulic resistance can be taken into account using the total hydraulic resistance coefficient ξ, so equation (3) was presented as:

$$\frac{\partial\bar{v}}{\partial\bar{t}}+\bar{v}\frac{\partial\bar{v}}{\partial\bar{x}}=-\frac{1}{\bar{\rho}}\frac{\partial\bar{p}}{\partial\bar{x}}-\xi\frac{\bar{v}^2}{2L}, \tag{4}$$

where $\xi=\xi_T+\xi_c$, $\xi_T=\lambda_T L/D_a$– friction loss coefficient in the pipe; ξ_c is the coefficient of local resistance at the point of connection to the volume. The coefficients ξ_T and ξ_c when calculating the flow in a pipe can be found approximately using the formulas used for stationary flows [31] if there is no reliable data on hydraulic resistance during flow pulsations.

Using dimensionless variables, the continuity and motion equations were obtained in dimensionless form:

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}\left(\rho v\right)=0, \tag{5}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=-\frac{1}{k_0\rho}\frac{\partial p}{\partial x}-\xi\frac{v^2}{2}, \tag{6}$$

where k_0=1.4 is the air adiabatic index.

For further transformations, the dimensionless rate of change of mass (or instantaneous mass flow) was introduced:

$$\frac{dm}{dt}=\frac{d\bar{m}_i/d\bar{t}}{\bar{\rho}_0\bar{a}_0 F}=-\mu_a\rho_a v_a, \tag{7}$$

where F is the characteristic cross-sectional area, μ_a is the flow coefficient at the entrance to the pipe, v_a, ρ_a are the speed velocity and density of the gas in the pipe.

If one takes the pipe length L as the characteristic size, then the cross-sectional area of the pipe F_a in some characteristic section can be taken as the characteristic area. At this stage of the study, it was considered appropriate to consider a pipe of constant cross-section, for which $F=F_a$.

Next, the assumption was made that the gas contained in the control volume under increased pressure flows through the pipe into the environment. From equation (6), using term-by-term multiplication by dx, one gets:

$$\frac{dv}{dt}dx+vdv=-\frac{1}{k_0\rho}dp-\xi\frac{v^2}{2}dx. \tag{8}$$

After integrating equation (8) from the center of the volume, where it can be approximately assumed that the gas is motionless, to the pipe cut, it took the form:

$$\int_{-x_K}^{1}\frac{dv}{dt}\mathrm{d}x+\int_{v_K}^{v_a}v\mathrm{d}v=-\frac{1}{k_0}\int_{p}^{p_a}\frac{\mathrm{d}p}{\rho}-\int_{-x_K}^{1}\xi\frac{v^2}{2}\mathrm{d}x, \tag{9}$$

where $x_K=\bar{x}_K/L$ is the relative distance from the nozzle exit to the center of the combustion chamber.

The second term of equation (9) after the adopted assumptions is equal to:

$$\int_{v_K}^{v_a}v\mathrm{d}v=\frac{v_a^2}{2}. \tag{10}$$

The last term of equation (9) is represented as:

$$\int_{-x_K}^{1}\xi\frac{v^2}{2}\mathrm{d}x=\xi_c\frac{v_c^2}{2}x_K+\xi_T\frac{v_a^2}{2}\approx\frac{v_a^2}{2}\left(\xi_T+x_K\xi_c\right). \tag{11}$$

Taking this into account, equation (9) is written as follows:

$$\int_{-x_K}^{1}\frac{dv}{dt}\mathrm{d}x=-\frac{1}{k_0}\int_{p}^{p_a}\frac{\mathrm{d}p}{\rho}-\frac{v_a^2}{2}K_\xi, \tag{12}$$

where K_ξ~$1+\xi_T+0.25\xi_c/\Lambda$.

Next, the first term of equation (12) was considered. For an incompressible gas, it follows from the continuity equation that vF=const. Consequently, the gas speed velocity in any section of the pipe is $v=v_aF_a/F$. From where, differentiating with respect to time, one gets:

$$\int_{-x_K}^{1}\frac{dv}{dt}\mathrm{d}x=\int_{-x_K}^{1}\frac{dv_a}{dt}\frac{F_a}{F}\mathrm{d}x=\frac{dv_a}{dt}\int_{-x_K}^{1}\frac{F_a}{F}\mathrm{d}x. \tag{13}$$

The quantity $\int_{-x_K}^{1}\frac{F_a}{F}\mathrm{d}x$, obviously:

$$\int_{-x_K}^{1}\frac{F_a}{F}\mathrm{d}x=1+\int_{-x_K}^{-x_c}\frac{F_a}{F}\mathrm{d}x+\int_{-x_c}^{0}\frac{F_a}{F}\mathrm{d}x=$$
$$=1+\frac{F_a}{F_K}\left(x_K-x_c\right)+\int_{-x_c}^{0}\frac{F_a}{F}\mathrm{d}x, \tag{14}$$

where x_c is the relative length of the transition part from the volume to the pipe.

The current diameter of the adapter, made, for example, in the form of a cone (Fig. 1), can be represented as:

$$D=D_a\frac{x_c+x}{x_c}-D_K\frac{x}{x_c}, \tag{15}$$

where D_K, D_a – transverse volume size and pipe diameter.

From where after transformations:

$$\int_{-x_c}^{0}\frac{F_a}{F}\mathrm{d}x=\int_{-x_c}^{0}\frac{\mathrm{d}x}{\left[1-\frac{x}{x_c}\left(\sqrt{\frac{F_K}{F_a}}-1\right)\right]^2}=\frac{x_c}{2+\sqrt{\frac{F_K}{F_a}}}. \tag{16}$$

For the completed designs of some types of engines with a periodic workflow [25], the area ratio is no less than F_K/F_a~3÷4. The volume of the transition part, if a relatively smooth transition is made, is usually somewhat smaller than the main volume. Based on this, it is assumed that the volume of the adapter is half the control volume, from where x_c can be written as:

$$x_c \approx \frac{0.5V}{(F_K + F_a)L} = \frac{1}{\Lambda(F_K / F_a + 1)} = \frac{0.175}{\Lambda},$$

where the dimensionless parameter $\Lambda = F_a L/V$ represents the relative volume of the pipe.

Since the value $0.05/\Lambda$ is quite small compared to 1, this made it possible to obtain an approximate equality from equation (17):

$$\int_{-x_K}^{1} \frac{F_a}{F} \mathrm{d}x \approx \frac{1}{1 - \frac{0.05}{\Lambda}}. \quad (17)$$

The resulting value is a certain correction factor to the length of the pipe, which allows one to take into account the size (length) of the volume from which the flow occurs into the pipe.

Thus, equation (12) was written as:

$$\frac{dv_a}{dt} \frac{1}{1 - \frac{0.05}{\Lambda}} = -\frac{1}{k_0} \int_{p}^{p_a} \frac{\mathrm{d}p}{\rho} - K_\xi \frac{v_a^2}{2}. \quad (18)$$

The final transformations were then made in the right-hand part of equation (18). Since the air that was in the pipe at the initial moment of time is pushed out by gases having a higher temperature, it is assumed that there is no mixing between gases and air. Then the air-gas boundary will move along the pipe and at some point in time it will move away from the beginning of the pipe to a distance x. There are gases with different densities in the pipe, so it was necessary to determine some average (reduced) gas density. At the same time, the gas density in the volume was also taken into account since equation (18) describes the movement of gas from the center of the volume to the pipe cut. The equation for the reduced density was written as:

$$\rho_{pr}(V + F_a L) = \rho V + \rho_x F_a x + \rho_0' F_a (1 - x), \quad (19)$$

where ρ_x is the density of gases in the pipe, ρ_0' is the density of air in the pipe.

Let ρ_{pr} be proportional to ρ over a certain period of time, i.e.: $\rho_{pr} = r\rho$, where r=const. Then:

$$r = \frac{\rho_{pr}}{\rho} = \frac{\Lambda}{1+\Lambda}\left[\frac{1}{\Lambda} + x\left(\frac{\rho_x}{\rho}\right) + (1-x)\left(\frac{\rho_0'}{\rho}\right)\right]. \quad (20)$$

The gas densities included in this equation are written using the ideal gas equation of state as:

$$\rho = \frac{p}{RT},\ \rho_x = \frac{p}{RT}\left(\frac{p_a}{p}\right)^{\frac{1}{k}},\ \rho_0' = \frac{p_0}{R_0 T_0}\left(\frac{p}{p_0}\right)^{\frac{1}{k_0}} = p^{\frac{1}{k_0}}. \quad (21)$$

In the absence of heat losses, it can be assumed that the gas density in the volume depends adiabatically on pressure. This made it possible to represent the corresponding term in equation (18) as:

$$-\frac{1}{k_0}\int_{p}^{p_a} \frac{\mathrm{d}p}{\rho} = -\frac{1}{k_0}\int_{p}^{p_a} \frac{\mathrm{d}p}{rCp^{\frac{1}{k}}} = \frac{k}{k-1}\frac{RT}{k_0}\frac{1}{r}\left[1 - \left(\frac{p_a}{p}\right)^{\frac{k-1}{k}}\right]. \quad (22)$$

After transforming equation (18) taking into account (20) to (22), one finally obtains an equation valid for $p \geq p_a$:

$$\frac{dv_a}{dt} = \left(1 - \frac{0.05}{\Lambda}\right) \times \times \left\{ \begin{array}{l} \frac{k}{k-1}\left(1 + \frac{1}{\Lambda}\right)\frac{RT}{k_0} \times \\ \times \frac{1 - \left(\frac{p_a}{p}\right)^{\frac{k-1}{k}}}{\frac{1}{\Lambda} + x\left(\frac{p_a}{p}\right)^{\frac{1}{k}} + (1-x)RT\left(\frac{1}{p}\right)^{\frac{k_0-1}{k_0}}} - K_\xi \frac{v_a^2}{2} \end{array} \right\}. \quad (23)$$

This equation is valid for the positive direction of the gas velocity in the pipe when the pressure in the control volume is greater than the static pressure at the pipe end. Similarly, an equation was obtained for the next section of the engine cycle with a periodic workflow, in which, with a constant direction of the flow velocity in the pipe, the pressure in the volume is less than the pressure at the pipe cut.

When considering the reversal of the direction of gas velocity in a pipe, it is noted that when integrating equation (9), the upper limits of integration should be changed to lower ones, and vice versa. Obviously, in this case, the signs of the corresponding terms of equation (18) should be changed, leaving the gas densities approximately equal.

For pressure in the volume above the static pressure at the pipe exit $p < p_a$, expressions for gas density similar to the previous case are obtained:

$$\frac{dv_a}{dt} = \left(1 - \frac{0.05}{\Lambda}\right) \times \times \left[\begin{array}{l} \left(1 + \frac{1}{\Lambda}\right)\frac{RT}{k_0} \times \\ \times \frac{\frac{p_a}{p} - 1}{\frac{1}{\Lambda} + x + (1-x)\frac{RT}{R_a T_a}\left(\frac{p_a}{p}\right)^{\frac{k_0-1}{k_0}}} - K_\xi \frac{v_a^2}{2} \end{array} \right]. \quad (24)$$

Equations for velocity (23) and (24) include the values of the coordinate x of the boundary separating gases and air with different densities, which obviously affects the mass and acceleration of the column in the pipe. To find this coordinate, one should consider the part of the pipe where the air is currently located. If one assumes that at the initial moment of time the air occupied the entire volume of the pipe, that is, $x=0$ at $t=t_0$, then when flowing from the volume, the coordinate can be easily found by numerical integration of the equation:

$$dx / dt = v_a. \quad (25)$$

If, during the outflow process, the value x found using equation (25) becomes greater than or equal to 1, this means that the air has been completely pushed out of the pipe. When the sign of the speed velocity changes, air will begin to flow into the pipe. Then at the moment the speed velocity sign changes $t=t_{01}$, $x=1$.

Integration of equations (23) to (25) is carried out numerically (when closing the 0-dimensional model, this is done together with the equations for pressure and temperature in the control volume). For this, it is quite acceptable to use the 2nd order Runge-Kutta method [32]. When checking the reliability of the model, the value of Λ included in the equations was taken in the 1st approximation to be equal to 1.5 (the volume of the resonant pipe is 1.5 times larger than the control volume to which the pipe is connected). Friction was not taken into account, that is, the coefficient K_ξ was assumed to be equal to 1.0. In addition, the flow of air of the same temperature was studied, therefore the current coordinate of the boundary x, included in equations (23) and (24), was not determined, but was also taken unchanged and equal to 1.0.

5. 2. Construction of an alternative 1-dimensional gas-dynamic model

To test the model, an alternative finite-difference 1-dimensional model of air flow in the same pipe was considered. In the 1st approximation, friction and heat losses were not taken into account since the solution was planned to be used only to test the method. In addition to the equations of continuity (2) and motion (3), such a problem requires an energy equation [33].

It was shown in [28] that using the equation of state, the energy equation can be transformed into a differential equation for pressure. In addition, since the piston analogy method is used to close the 0-dimensional model of changes in gas parameters in a control volume, it is quite logical to calculate the same parameters for the pipe, that is, pressure and temperature. Therefore, the continuity equation (2) using the equation for pressure was transformed into an equation for temperature.

As a result, the system of equations describing the 1-dimensional movement of air in a pipe was represented in the following dimensionless form:

$$\begin{cases} \dfrac{dv}{dt} = -v\dfrac{dv}{dx} - \dfrac{1}{\rho}\dfrac{dp}{dx}, \\ \dfrac{dp}{dt} = -v\dfrac{dp}{dx} - kp\dfrac{dv}{dx}, \\ \dfrac{dT}{dt} = -v\dfrac{dT}{dx} - (k-1)T\dfrac{dv}{dx}. \end{cases} \quad (26)$$

The initial conditions were set in the form $p=1$, $T=1$, $v=0$for all points of the pipe. The boundary conditions were adopted as follows. At the left end of the pipe (($i=0$) $p=p_0$, $T=T_0$, $v=0$, at the right end of the pipe ($i=N+1$) $p_{N+1}=1$, $T_{N+1}=T_N$, $v_{N+1}=v_N$ at $v_N>0$ and $T_{N+1}=1$, $v_{N+1}=0$ at $v_N \leq 0$ (Fig. 2).

The solution was performed by the Lax-Wendroff method [28, 32] in the form of forward differences in time and central differences in coordinate for the variables v, p, and T (written below as ξ):

$$\frac{\xi_i^{n+1}-\xi_i^n}{\Delta t} = -\frac{v_i}{2\Delta x}\left(\xi_{i+1}^n - \xi_{i-1}^n\right) + \\ +\frac{1}{2}v_i^2\frac{\Delta t}{\Delta x^2}\left(\xi_{i+1}^n + \xi_{i-1}^n - 2\xi_i^n\right). \quad (27)$$

The division of the step Δt into time and coordinate Δx was carried out based on the condition of ensuring the Courant number C less than 1.0 [28], however, during testing, the solution was stable at a value C less than 0.4. As a result, calculations were carried out with a dimensionless time step of 0.05 (400 steps) and a dimensionless coordinate of 0.05 (dividing the pipe into 20 segments).

To assess the correct operation of the program for the 1D model, data from similar calculations or experiments were not used since the task was previously assessed as quite simple. In this regard, the reliability of the results was checked only for visual qualitative correspondence to the nature of the flow under study.

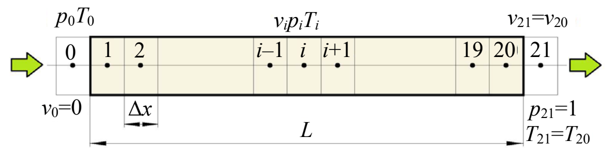

Fig. 2. Diagram of a pipe in a one-dimensional flow model

5. 3. Mathematical modeling of air movement in a pipe using both methods based on the data obtained

In the verification calculation using the 1D model, a single pulse was set at the entrance to the resonant tube $p_0=1.3$ for 10 time steps, after which the pressure at the entrance was taken equal to $p=1$. The time results are shown in Fig. 3 for 3 sections – at the entrance, in the middle, and at the exit from the pipe, and by coordinate – for a different number of time steps.

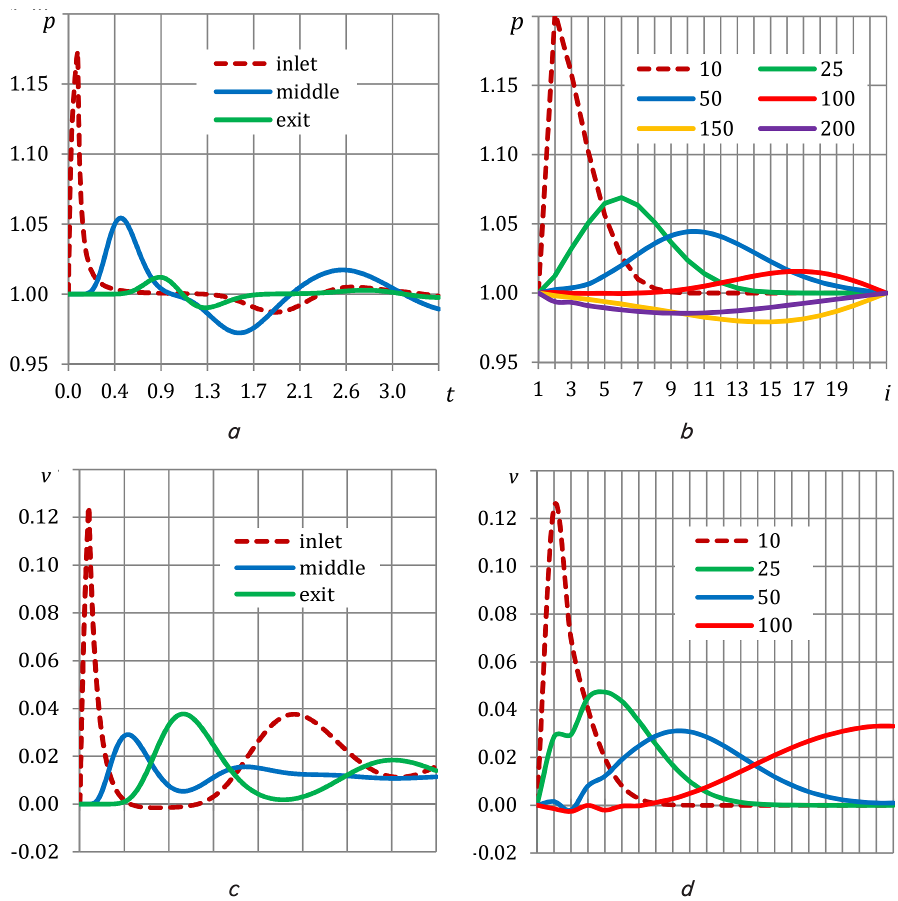

Fig. 3. Calculated using a 1D model, the change in pressure and speed velocity in the pipe in time and coordinate from the pressure pulse at the end of the pipe: *a, c* – pressure and speed velocity at the ends and in the middle of the pipe; *b, d* – distribution of pressure and velocity along the pipe over time (by the number of time steps)

Next, two comparative tests were performed.

Test No. 1: constant differential pressure. The instantaneous opening of the left end of the pipe was simulated, into which air enters under constant pressure p_0=1.3 in the adjacent volume (Fig. 4).

Test No. 2: the pressure drop varies sinusoidally from minimum to maximum, which simulates the corresponding elements of an engine with a periodic workflow (Fig. 5).

The results of test No. 2 are also summarized in Table 1 in the form of the difference (error) of the piston analogy model in comparison with the finite-difference model.

Table 1

Comparison of simulation results for instantaneous and integral parameters at different process frequencies

Parameter	Instantaneous air speed velocity		Air consumption per operating cycle	
Dimensionless process frequency, *f*	0.29	0.58	0.29	0.58
Piston analogy model error, %	+5.0 %	+20 %	+4.5 %	+2.5 %

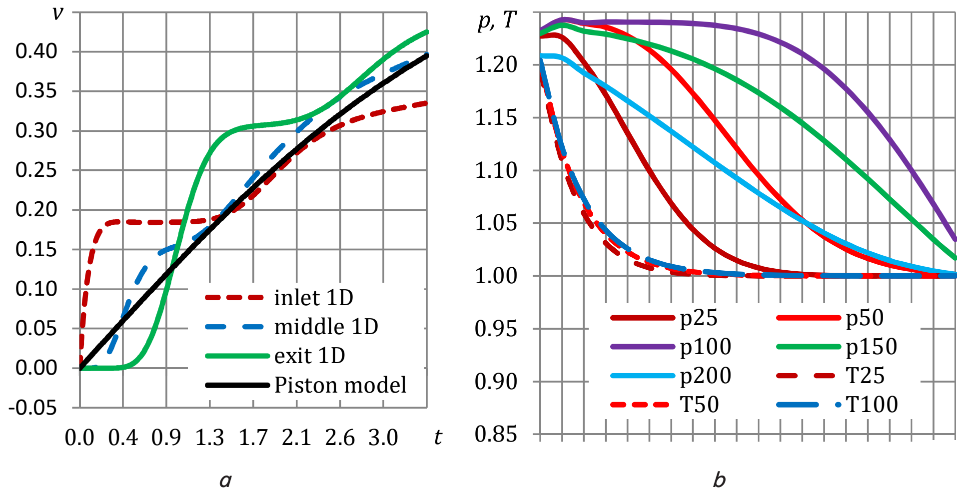

Fig. 4. The flow process in a pipe at a constant pressure drop at p_0=1.3 after opening the left end of the pipe: *a* – velocities at the ends and in the middle of the pipe in comparison with the speed velocity calculated using the piston model; *b* – distribution of pressure *p* and temperature *T* along the length of the pipe at different times (by the number of points in time)

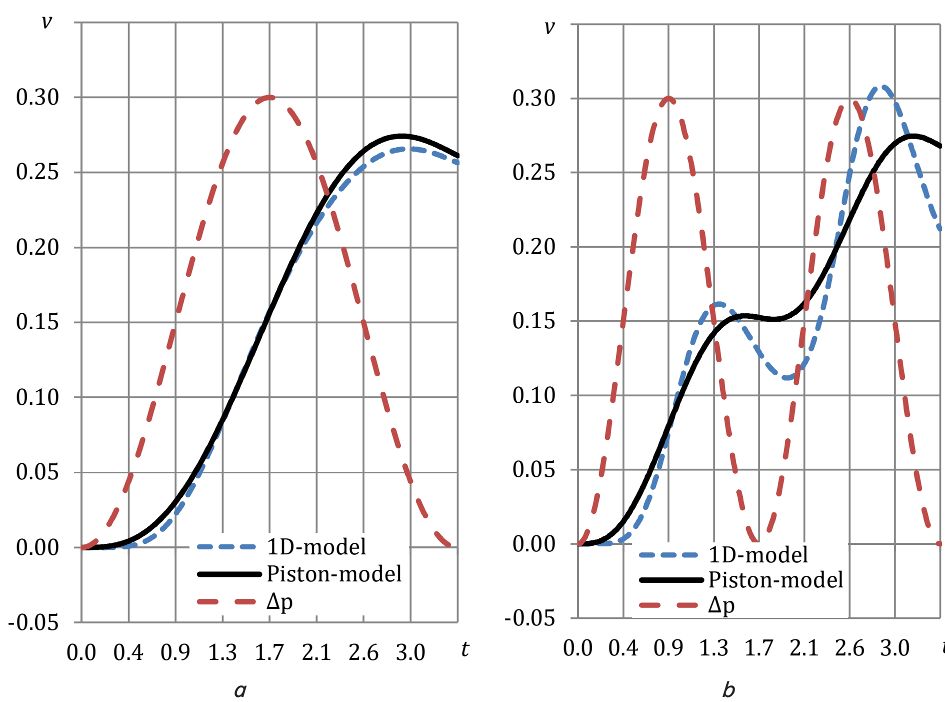

Fig. 5. The speed velocity of air flow in the middle of the pipe when the pressure drop at the inlet changes according to a sinusoidal law: *a* – with a dimensionless pulsation frequency *f*=0.29; *b* – at double frequency *f*=0.58

The above data show generally satisfactory convergence of the results obtained using the piston analogy model for both instantaneous speed velocity and integral air flow per cycle.

5. 4. Comparative analysis of the accuracy, reliability, and limits of applicability of the piston analogy model

As follows from the curves (Fig. 3) obtained using the finite-difference model, a pressure pulse at the left end of the pipe leads to the formation and propagation of a pressure wave along the pipe. The wave reflected from the free (right) end of the pipe passes to the left in the form of a rarefaction wave (Fig. 3, *a*, *b*). In this case, the air speed velocity gradually lags behind the pressure in length and time (Fig. 3, *c*, *d*). That is, the results are generally consistent with the physics of the process [34] (the slight instability of the velocity calculation at the beginning of the pipe, which can be seen in Fig. 3, *d*, was considered insignificant). This allowed the use of a finite difference model to test the piston model.

The case of opening the left end of the pipe to connect to a control volume with increased pressure shows that after the transient process a steady flow is established – the beginning of the process is shown in Fig. 4. However, the greatest interest is in the desired middle of the pipe, where the change in speed velocity calculated using the piston model coincided with the result of the calculation using the 1D model with an accuracy of no worse than 10 %.

As follows from Fig. 5, *a*, with a sinusoidal change in pressure in the volume, the velocity in the middle of the pipe noticeably lags behind the pressure in phase, but the difference between the models under consideration is minimal and amounts to no more than 5 %. This indicates that the speed velocity of air flow, calculated as the movement of the column mass under the influence of pressure and inertia forces, is practically no different from the speed velocity obtained as a result of the propagation of pressure waves. Accordingly, the difference in air flow through the pipe will also be minimal, and it is the air flow from the control volume that is the purpose of the calculation in this case.

However, this is only true at relatively low frequencies of forced oscillations (motor engine operating frequencies). As the frequency of pressure oscillations increases, the error in calculating the speed velocity of the piston model increases (Fig. 5, *b*), and in some parts of the pulsating cycle reaches 20 %.

A similar result of an increase in the error in calculating the speed velocity for the piston model should be expected with increasing pipe length. This is quite natural since when deriving the estimation equations, assumptions were made that were valid only for relatively short pipes. In addition, when operating as part of an engine, processes may be more abrupt than the sinusoid used during testing. In other words, since the instantaneous air flow will follow the change in speed velocity, an error in its calculation will affect the state of the gas in the control volume. However, calculating the total air flow rate per cycle for the selected example gives an even smaller error at an increased process frequency (Table 1). This confirms that for similar tasks, as well as when developing and debugging general models of engines of the type under consideration, it is permissible to use simpler models of elements without significant damage to accuracy.

Within the framework of the dimensionless time $t=\bar{t}\bar{a}_0/L$ adopted in deriving the equations, the dimensionless frequency (equal to the Strouhal number of the process under study [35]) is the inverse of the cycle time and is, according to Fig. 5, *a*, $f=\bar{f}L/\bar{a}_0=0.29$. Based on this, the error value for a speed velocity of 5 % is taken as the maximum permissible, from where, with a known speed of sound velocity for air, the parameter $\left(\bar{f}L\right)_{\max}\approx 100.$ can be obtained. This value, which has the dimensionality of speed velocity (m/s), sets a limitation on the use of the piston model depending on the dimensional factor. For example, at a process frequency of 150 Hz, according to the limitation, the maximum length of the pipe adjacent to the control volume of the intermittent engine should not exceed 0.7 m. Otherwise, for longer pipes and/or higher frequencies, one should proceed to traditional 1D or higher dimensional flow models.

6. Discussion of results of modeling air flow in a pipe

From the results of theoretical studies, it is clearly seen that solving a differential equation for instantaneous velocity averaged over the length of a pipe is much simpler than solving a system of 3 differential equations for velocity, pressure, and temperature. At the same time, attention is drawn to the need to additionally determine the coordinates of the moving boundary for the case of a pulsating flow of hot gas, which occurs automatically when solving a system of equations in a 1-dimensional statement. Logic is also required to select the equations of motion of the "liquid" piston when changing the flow direction, which in the case of a system of equations of 1-dimensional flow is carried out only by changing the boundary conditions.

The test using a single pulse (Fig. 3) was performed to check the stability and convergence of the solution to the system of equations for 1-dimensional air flow in a pipe. This test is also important because it shows the acceptability of using the form of writing the system of equations (26) in which instead of equations for momentum and energy, equations for pressure and temperature are used. As a result of the test, stable propagation of pressure and rarefaction waves in a pipe with attenuation due to artificial circuit viscosity was obtained, which is consistent with the known data and principles of constructing difference circuits [28]. At the same time, a restriction on the Courant number on the relationship between the time step and coordinate was found ($C<0.4$), which was used in subsequent tests.

The constant pressure drop test (Fig. 4) was carried out to determine the limit to which the flow velocity tends after the left end of the pipe is instantly opened. In fact, the test shows the difference in the dynamic properties of air in the representation of a "liquid" piston and a compressible medium. According to Fig. 4, *a*, this difference for the considered pipe parameters is insignificant, which provides additional arguments in favor of the possibility of using the concept of a "liquid" piston in modeling practice.

The results of modeling pulsating air flow in a pipe using the proposed method (Fig. 5, Table 1) show that the error in calculating speed velocity and flow rate under certain conditions can be small. Physically, this result corresponds to a small mass of the gas column in the pipe (low inertia of the "liquid" piston). This means that when developing new models of intermittent engines, relatively simple models can be used. This can be especially important at the stage of debugging algorithms and programs. At the same time, it is not at all necessary to immediately strive for the most accurate physical models, and even more so, to increase their dimensionality. This is exactly how the proposed piston analogy model is constructed. Moreover, unlike many other models, not only a fairly simple and effective model is proposed but the permissible limits of its application were separately studied and indicated in comparison with the most frequently used ones. The result was a more complete and objective picture of the feasible scope of the model, something that is usually neglected in higher-dimensional models.

In known models, to obtain the air (gas) flow rate, a numerical solution of a system of differential equations is required, and sequentially for each cell (volume) into which the pipe is divided. And this is tens and hundreds of times at each time step. On the contrary, the main advantage of the proposed model is the ability to calculate the air (gas) flow rate through a pipe under unsteady flow conditions by numerically solving in time only one differential equation for velocity. This allows one to significantly simplify the model, as well as reduce the time and labor intensity of debugging any new models and programs for calculating the operating cycle of engines with a periodic workflow with a slight loss of accuracy.

At the same time, the proposed model, developed on the basis of simplifying assumptions, has not been tested in a wide range of changes in the parameters and modes of engines with a periodic workflow. Also, a specific linkage of the model to engine types and their elements has not yet been carried out. In addition, when constructing the model, in order to simplify it, the influence of heat transfer on the flow in the resonant tube was not taken into account. This assumption does not cause a fundamental error in the modeling but for some engines and components, especially the hot circuit, the error may be noticeable.

As a consequence, it is expected that in further studies, in order to refine the model, the influence of heat transfer on the nature of the flow in the pipe will be considered. In addition, it is necessary to study the model with a variable pipe cross-section in order to better adapt it to the configuration features of real structures. This will make it possible to refine the model and make it better applicable to work as part of general models of a wider range of engines of the type under consideration.

7. Conclusions

1. A simple piston analogy model has been built to describe the unsteady gas flow in a pipe adjacent to the control volume (cylinder, combustion chamber) and to close the thermodynamic models of the engine cycle with a periodic workflow. Unlike the known ones, the model is implemented by numerically solving only one differential equation for the instantaneous gas velocity, previously averaged over the length of the pipe.

2. To test the model built, an alternative finite-difference 1-dimensional gas-dynamic model of air flow in a pipe was selected, which was adapted to solve the problem. Unlike the known ones, this model solves numerically a system of differential equations for the instantaneous velocity, pressure, and temperature of the gas along the length of the pipe, rather than velocity, momentum, and energy. This representation better corresponds to a 0-dimensional thermodynamic model that describes changes in gas pressure and temperature in the control volume of an engine with a periodic workflow.

3. Using the constructed models, control and test modeling of unsteady air flow in a pipe was performed under various modes. Test modes were selected in accordance with the parameters of certain types of engines with periodic workflow, in which pressure drops in the resonant tubes are small. Thus, the amplitude of pressure fluctuations, including a short pressure pulse at the inlet, as well as a constant and pulsating sinusoidal pressure drop, was limited in testing modes to a dimensionless pressure value of 1.3.

4. Based on the data obtained, a comparative analysis of the accuracy and reliability of the piston analogy model was carried out. According to the simulation results, the piston model allows one to find the flow velocity in the pipe with an accuracy of 5 % for a pressure drop varying according to a sinusoidal law. The permissible limits for changes in the oscillation frequency and pipe length, determined by the maximum value of the Strouhal number of 0.29, were found, at which the piston model has a minimal error compared to the 1-dimensional model. It has been established that with proper consideration of the existing limitations, the piston model gives results close to those that are provided by more complex models with higher dimensionality. This indicates the possibility of using a piston model for elements such as pipes as part of a 0-dimensional thermodynamic model of an engine with a periodic workflow as an approximate alternative to traditional 1-dimensional flow models.

Conflicts of interest

The author declares that he has no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

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